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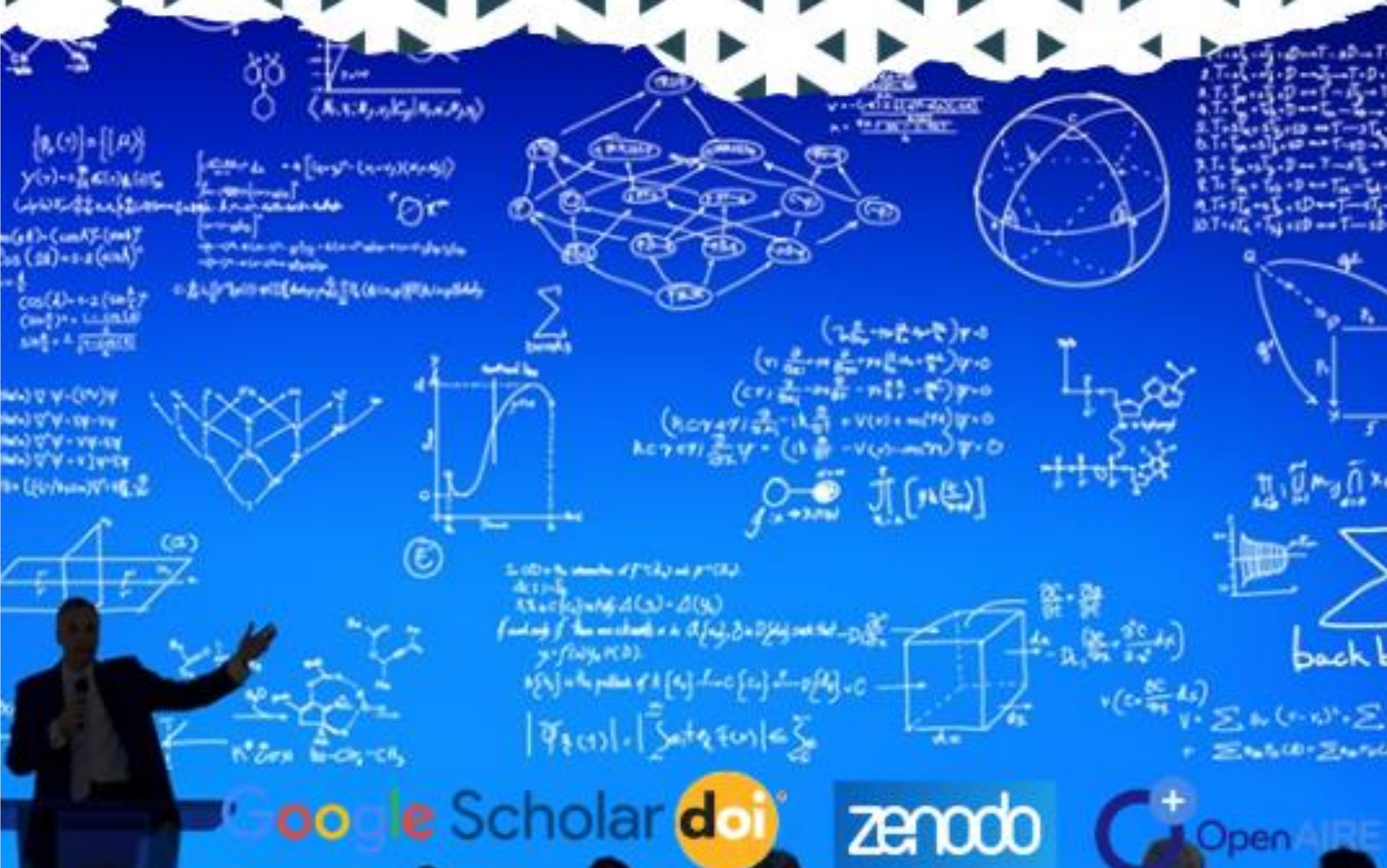
ILMIY-AMALIY KONFERENSIYA



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Ko'p o'zgaruvchili funksiyaning differensiali.

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Annotasiya. Ko'p o'zgaruvchili funksiyaning differensiali mavzusi matematik analizning muhim bo'limlaridan biri hisoblanadi. Ushbu mavzu ko'p o'zgaruvchili funksiyalarning o'zgarish qonuniyatlarini, ularning lokal xossalarini hamda yaqinlashtirish usullarini o'rganishga qaratilgan. Differensial tushunchasi orqali funksiyaning kichik o'zgarishlaridagi taxminiy qiymatlari aniqlanadi va bu esa turli amaliy masalalarni echishda muhim ahamiyat kasb etadi.

Mazkur ishda ko'p o'zgaruvchili funksiyaning qisman hosilalari, to'liq differensiali va uning geometrik hamda fizik ma'nolari yoritiladi. SHuningdek, differensialning qo'llanilishi orqali funksiyalarning ekstremumlarini topish, xatoliklarni baholash va optimallashtirish masalalarini hal etish imkoniyatlari ko'rsatib beriladi.

Ushbu mavzu talabalarda analitik fikrlashni rivojlantirish, murakkab jarayonlarni modellashtirish va amaliy masalalarni samarali hal etish ko'nikmalarini shakllantirishda muhim o'rin tutadi.

Kalit so'zlar: ko'p o'zgaruvchili funksiya, differensial, qisman hosila, to'liq differensial, funksiya o'zgarishi, yaqinlashtirish, ekstremum, optimallashtirish, gradient, lokal xossalar, matematik analiz, xatolikni baholash

Adabiyotlar tahlili. Mazkur mavzuni o'rganishda E. Jummayevning ilmiy ishlari va o'quv qo'llanmalari muhim manba sifatida xizmat qiladi. Uning tadqiqotlarida ko'p o'zgaruvchili funksiyalar nazariyasi, xususan, qisman hosilalar va to'liq differensial tushunchalari izchil va tushunarli tarzda bayon etilgan.

E. Jummayev o'z asarlarida differensialning nazariy asoslarini chuqur yoritib, uning geometrik va amaliy ahamiyatini keng misollar orqali tushuntirib beradi. SHu bilan birga, muallif ko'p o'zgaruvchili funksiyalarning ekstremumlarini topish, gradient va yo'nalishli hosilalar kabi masalalarni ham batafsil tahlil qiladi.

Adabiyotlar tahlili shuni ko'rsatadiki, E. Jummayevning ishlari talabalar va tadqiqotchilar uchun qulay uslubda yozilgan bo'lib, ular ko'p o'zgaruvchili funksiyalar differensiali mavzusini chuqur o'zlashtirishda muhim ahamiyat kasb etadi.

Tadqiqot usuli. Mazkur mavzuni o'rganish jarayonida bir qator ilmiy tadqiqot usullaridan foydalanildi. Jumladan, nazariy tahlil usuli orqali ko'p o'zgaruvchili funksiyaning differensialiga oid ilmiy adabiyotlar o'rganildi va umumlashtirildi. Taqqoslash usuli yordamida turli mualliflar, xususan, E. Jummayev ishlarida keltirilgan yondashuvlar o'zaro solishtirildi.

SHuningdek, matematik modellashtirish usuli orqali ko'p o'zgaruvchili funksiyalarning o'zgarish qonuniyatlari tahlil qilindi hamda differensial tushunchasining amaliy qo'llanilishi ko'rib chiqildi. Analitik usul yordamida qisman hosilalar va to'liq differensial formulalari chiqarildi va izohlandi.

Tadqiqot jarayonida, shuningdek, misollar asosida tushuntirish (illyustrativ) usulidan ham foydalanilib, nazariy bilimlarni mustahkamlashga alohida e'tibor qaratildi. Bu esa mavzuni chuqur va har tomonlama anglash imkonini berdi.

Tahlil va natijalar:

Faraz qilaylik, $f(x)=f(x_1,x_2,\ldots,x_m)$ funksiya $E\subset R^m$ da berilgan bo'lib, $x^0=\left(x_1^0,x_2^0,\ldots,x_m^0\right)\in E$ nuqtada differensiallanuvchi bo'lsin. Unda ta'rifga ko'ra funksiyaning x^0 nuqtadagi to'liq orttirmasi

$$\Delta f\left(x^0\right)=\frac{\partial f\left(x^0\right)}{\partial x_1}\Delta x_1+\frac{\partial f\left(x^0\right)}{\partial x_2}\Delta x_2+\cdots+\frac{\partial f\left(x^0\right)}{\partial x_m}\Delta x_m+o(\rho) \qquad (1)$$

bo'ladi. Bu munosabatda

$$\rho=\sqrt{\Delta x_1^2+\Delta x_2^2+\cdots+\Delta x_m^2}$$

bo'lib, $\Delta x_1\to 0$, $\Delta x_2\to 0$, … , $\Delta x_m\to 0$ da $\rho\to 0$.

1-ta'rif. $f(x)$ funksiyaning $\Delta f\left(x^0\right)$ orttirmasidagi

$$\frac{\partial f\left(x^0\right)}{\partial x_1}\Delta x_1+\frac{\partial f\left(x^0\right)}{\partial x_2}\Delta x_2+\cdots+\frac{\partial f\left(x^0\right)}{\partial x_m}\Delta x_m$$

ifoda $f(x)$ funksiyaning x^0 nuqtadagi differensiali (to'liq differensiali) deyiladi va

$$df\left(x^0\right) \text{ yoki } df\left(x_1^0,x_2^0,\ldots,x_m^0\right)$$

kabi belgilanadi:

$$df\left(x^0\right)=\frac{\partial f\left(x^0\right)}{\partial x_1}\Delta x_1+\frac{\partial f\left(x^0\right)}{\partial x_2}\Delta x_2+\cdots+\frac{\partial f\left(x^0\right)}{\partial x_m}\Delta x_m.$$

Demak, $f(x)$ funksiyaning x^0 nuqtadagi differensiali $\Delta x_1,\Delta x_2,\ldots,\Delta x_m$ larga bog'liq va ularning chiziqli funksiyasi bo'ladi.

Agar

$$\Delta x_1=dx_1,\ \Delta x_2=dx_2,\ \ldots,\ \Delta x_m=dx_m$$

deyilsa, $f(x)$ funksiyaning x^0 nuqtadagi differensiali ushbu

$$df\left(x^0\right)=\frac{\partial f\left(x^0\right)}{\partial x_1}dx_1+\frac{\partial f\left(x^0\right)}{\partial x_2}dx_2+\cdots+\frac{\partial f\left(x^0\right)}{\partial x_m}dx_m \qquad (2)$$

ko'rinishga keladi. Demak,

$$\Delta f\left(x^0\right)=df\left(x^0\right)+0(\rho).$$

Keyingi tenglikdan $\rho\to 0$ da

$$\Delta f\left(x^0\right)\approx df\left(x^0\right)$$

bo'lishi kelib chiqadi. Bu taqribiy formulaning mohiyati shundaki, funksiyaning orrtirmasi $\Delta x_1,\Delta x_2,\ldots,\Delta x_m$ larning, umuman aytganda murakkab funksiyasi bo'lgan holda funksiyaning differensiali $\Delta x_1,\Delta x_2,\ldots,\Delta x_m$ larning chiziqli funksiyai bo'lishidadir.

2^0. Murakkab funksiyaning differensiali. Differen-sial shaklning invariantligi. Aytaylik,

$$x_1 = \varphi_1(t) = \varphi_1(t_1, t_2, \ldots, t_k),$$
$$x_2 = \varphi_2(t) = \varphi_1(t_1, t_2, \ldots, t_k),$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$x_m = \varphi_m(t) = \varphi_1(t_1, t_2, \ldots, t_k)$$

funksiyalarning har biri $M \subset R^k$ to'plamda berilgan bo'lib,

$$E = \{(x_1, x_2, \ldots, x_m) \in R^m : x_1 = \varphi_1(t) = \varphi_1(t_1, t_2, \ldots, t_k),$$
$$x_2 = \varphi_2(t) = \varphi_1(t_1, t_2, \ldots, t_k), \ldots, x_m = \varphi_m(t) = \varphi_1(t_1, t_2, \ldots, t_k)\}$$

to'plamda esa $f(x_1, x_2, ..., x_m)$ funksiya aniqlangan bo'lsin. Bular yordamida

$$f(x(t)) = f(x_1(t), x_2(t), ..., x_m(t)) = F(t_1, t_2, ..., t_k)$$

murakkab funksiya hosil qilingan bo'lsin.

Ma'lumki, $x_i = \varphi_i(t_1, \ldots, t_k)$ funksiyalar $(i = 1, 2, \ldots m)$ $t^0 = (t_1^0, \ldots, t_k^0)$ nuqtada differensiallanuvchi bo'lib, $f(x) = f(x_1, x_2, \ldots, x_m)$ funksiya mos $x^0 = (x_1^0, x_2^0, \ldots, x_m^0)$ nuqtada $(x_1^0 = \varphi_1(t^0), x_2^0 = \varphi_2(t^0), \ldots, x_m^0 = \varphi_m(t^0))$ differensiallanuvchi bo'lsa, murakkab funksiya $t^0 = (t_1^0, \ldots, t_k^0)$ nuqtada differensialanuvchi bo'ladi.

Modomiki, $f(x(t))$ funksiya $t_1, t_2, \ldots, t_k$ o'zgaruvchilarga bog'liq ekan, unda

$$df = \frac{\partial f}{\partial t_1} dt_1 + \frac{\partial f}{\partial t_2} dt_2 + \cdots + \frac{\partial f}{\partial t_m} dt_m \qquad (3)$$

bo'ladi.

Murakkab funksiyaning xususiy hosilalarini hisoblash formulalaridan foydalanib topamiz:

$$\frac{\partial f}{\partial t_1} = \frac{\partial f}{\partial x_1}\frac{\partial x_1}{\partial t_1} + \frac{\partial f}{\partial x_2}\cdot\frac{\partial x_2}{\partial t_1} + \ldots + \frac{\partial f}{\partial x_m}\cdot\frac{\partial x_m}{\partial t_1},$$

$$\frac{\partial f}{\partial t_2} = \frac{\partial f}{\partial x_1}\frac{\partial x_1}{\partial t_2} + \frac{\partial f}{\partial x_2}\frac{\partial x_2}{\partial t_2} + \ldots + \frac{\partial f}{\partial x_m}\cdot\frac{\partial x_m}{\partial t_2},$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\frac{\partial f}{\partial t_k} = \frac{\partial f}{\partial x_1}\frac{\partial x_1}{\partial t_k} + \frac{\partial f}{\partial x_2}\cdot\frac{\partial x_2}{\partial t_k} + \ldots + \frac{\partial f}{\partial x_m}\cdot\frac{\partial x_m}{\partial t_k}.$$

Bu xususiy hosilalarni (3) ifodadagi $\frac{\partial f}{\partial t_1}, \frac{\partial f}{\partial t_2}, \ldots, \frac{\partial f}{\partial t_k}$ larning o'rniga qo'yamiz. Natijada

$$df = \left[\frac{\partial f}{\partial x_1}\cdot\frac{\partial x_1}{\partial t_1} + \frac{\partial f}{\partial x_2}\cdot\frac{\partial x_2}{\partial t_1} + \cdots + \frac{\partial f}{\partial x_m}\cdot\frac{\partial x_m}{\partial t_1}\right] dt_1 +$$
$$+ \left[\frac{\partial f}{\partial x_1}\cdot\frac{\partial x_1}{\partial t_2} + \frac{\partial f}{\partial x_2}\cdot\frac{\partial x_2}{\partial t_2} + \cdots + \frac{\partial f}{\partial x_m}\cdot\frac{\partial x_m}{\partial t_2}\right] dt_2 +$$

$$+\cdots+\left[\frac{\partial f}{\partial x_1}\cdot\frac{\partial x_1}{\partial t_k}+\frac{\partial f}{\partial x_2}\cdot\frac{\partial x_2}{\partial t_k}+\cdots+\frac{\partial f}{\partial x_m}\cdot\frac{\partial x_m}{\partial t_k}\right]dt_k=$$

$$=\frac{\partial f}{\partial x_1}\left[\frac{\partial x_1}{\partial t_1}dt_1+\frac{\partial x_1}{\partial t_2}dt_2+\cdots+\frac{\partial x_1}{\partial t_k}dt_k\right]+$$

$$+\frac{\partial f}{\partial x_2}\left[\frac{\partial x_2}{\partial t_1}dt_1+\frac{\partial x_2}{\partial t_2}dt_2+\cdots+\frac{\partial x_2}{\partial t_k}dt_k\right]+\cdots+\frac{\partial f}{\partial x_m}\left[\frac{\partial x_m}{\partial t_1}dt_1+\frac{\partial x_m}{\partial t_2}dt_2+\cdots+\frac{\partial x_m}{\partial t_k}dt_k\right]$$

bo'ladi.

Ravshanki,

$$\frac{\partial x_1}{\partial t_1}dt_1+\frac{\partial x_1}{\partial t_2}dt_2+\ldots+\frac{\partial x_1}{\partial t_k}dt_k=dx_1,$$

$$\frac{\partial x_2}{\partial t_1}dt_1+\frac{\partial x_2}{\partial t_2}dt_2+\ldots+\frac{\partial x_2}{\partial t_k}dt_k=dx_2,$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\frac{\partial x_m}{\partial t_1}dt_1+\frac{\partial x_m}{\partial t_2}dt_2+\ldots+\frac{\partial x_m}{\partial t_k}dt_k=dx_k.$$

Demak, murakkab funksiyaning differensiali

$$df=\frac{\partial f}{\partial x_1}dx_1+\frac{\partial f}{\partial x_2}dx_2+\ldots+\frac{\partial f}{\partial x_m}dx_m \quad (4)$$

bo'ladi.

Biz yuqorida $f(x)$ hamda $f(x(t))$ murakkab funksiyaning differensallari uchun (2) va (4) ifodalarni topdik. Bu ifodalarni solishtirib ularning formasi (shakli, ko'rinishi) bir xil, ya'ni (2) va (4) formulalarda funksiyaning differensiali xususiy hosilalarni mos differensiallarga ko'paytmalardan tuzilgan yig'indiga teng ekanligini payqaymiz. Bu xossa differensial shaklning **invariantligi** deyiladi.

Eslatma. $f(x)$ funksiya differensialining (2) ifodasi-dagi $dx_1,dx_2,\ldots,dx_m$ lar mos ravishda $\Delta x_1,\Delta x_2,\ldots,\Delta x_m$ lar bo'lsa, $f(x(t))$ funksiya differensialidagi $dx_1,dx_2,\ldots,dx_m$ lar $t_1,t_2,\ldots,t_k$ o'zgaruv-chilarning funksiyalari bo'ladi. Demak, (2) va (4) formula-larning ko'rinishlarigina bir xil bo'ladi.

3^0. Sodda qoidalar. Aytaylik,

$$u=u(x_1,x_2,\ldots,x_m),\ \ v=v(x_1,x_2,\ldots,x_m)$$

funksiyalari $E\subset R^m$ to'plamda berilgan bo'lib, $x^\circ=(x_1^\circ,x_2^\circ,\ldots,x_m^\circ)\in E$ nuqtada differensiallanuvchi bo'lsin. U holda:

$$1)\ d(u+v)=du+dv,$$

$$2)\ d(u\cdot v)=vdu+udv,$$

$$3)\ d\left(\frac{u}{v}\right)=\frac{vdu-udv}{v^2} \qquad (v\neq 0)$$

bo'ladi.

Bu munosabatlardan birini, masalan, 3) ning isbotini keltiramiz.

◀Aytaylik,

$$F = \frac{u}{v}$$

bo'lsin. Bu holda F funksiya u va v larga va u va v lar o'z navbatida $x_1, x_2, ..., x_m$ o'zgaruvchilarga bog'liq bo'lib, murakkab funksiyaga ega bo'lamiz. Differensial shaklning invariant-li xossasiga ko'ra

$$dF = \frac{\partial F}{\partial u}du + \frac{\partial F}{\partial v}dv$$

bo'ladi. Ravshanki,

$$\frac{\partial F}{\partial u} = \frac{1}{v}, \frac{\partial F}{\partial v} = -\frac{u}{v^2}$$

Demak,

$$dF = \frac{1}{v}du - \frac{u}{v^2}dv = \frac{vdu - ydv}{v^2},$$

ya'ni

$$d\left(\frac{u}{v}\right) = \frac{vdu - udv}{v^2}$$

bo'ladi.

Xulosa. Ko'p o'zgaruvchili funksiyaning differensiali mavzusi matematik analizning muhim va zarur bo'limlaridan biri hisoblanadi. Ushbu mavzuda funksiyalarning bir nechta o'zgaruvchilarga bog'liq holda qanday o'zgarishi, ularning lokal xossalari va yaqinlashtirish imkoniyatlari chuqur o'rganildi. Qisman hosilalar va to'liq differensial tushunchalari orqali funksiyaning kichik o'zgarishlardagi xulq-atvori aniqlanishi ko'rsatib berildi.

Tadqiqot natijalari shuni ko'rsatadiki, differensial tushunchasi nafaqat nazariy ahamiyatga ega, balki amaliy masalalarni echishda ham keng qo'llaniladi. Xususan, ekstremumlarni topish, optimallashtirish, xatoliklarni baholash kabi jarayonlarda uning o'rni beqiyosdir. Bu borada E. Jummayev ishlari muhim ilmiy asos sifatida xizmat qiladi.

Xulosa qilib aytganda, ko'p o'zgaruvchili funksiyaning differensiali mavzusini puxta o'zlashtirish talabalarda mantiqiy va analitik fikrlashni rivojlantiradi hamda kelgusida murakkab matematik va amaliy masalalarni samarali hal etishga zamin yaratadi.

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