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## SOME CARDINAL PROPERTIES OF THE NEMYTSKY PLANE

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**Abstract.** this article investigates the cardinal and topological properties of the Nemytsky plane. It is known that this plane is a separable space without a countable base, so it is not a metrizable space. This plane satisfies the first axiom of countability, and does not satisfy the second axiom of countability. We present some other interesting properties of the Nemytsky plane. The following cardinal invariants of the Nemytsky plane are found this paper: tightness, network weight, pseudocharacter, spread, extent, local dense, functional and weak tightness, preshannumber,  $\pi$ -character, Shannumber, number of open sets. Other proofs are given about the Nemytsky plane is not normal space, about the uncountability of the weight, and about the countability of the character of the Nemytsky plane.

**Key words:** topological space; Nemytsky plane; tightness, network weight, pseudocharacter, spread, extent, local dense, functional and weak tightness, preshannumber,  $\pi$ -character, Shannumber.

**Anotatsiya.** Ushbu maqolada Nemitskiy tekisligining kardinal va topologik xususiyatlari o'rganiladi. Ma'lumki, bu tekislik hisoblanuvchi asossiz bo'linadigan fazodir, shuning uchun u o'chanadigan fazo emas. Bu tekislik sanashning birinchi aksiomasini qanoatlantiradi, ikkinchi aksiomani esa qanoatlantirmaydi. Biz Nemitskiy samolyotining boshqa qiziqarli xususiyatlarini taqdim etamiz. Ushbu maqolada Nemytskiy tekisligining quyidagi kardinal invariantlari topilgan: zichlik, tarmoq og'irligi, psevdokarakter, tarqalish, kenglik, mahalliy zichlik, funktsional va zaif zichlik, preshansoni, -belgi, Shansoni, ochiq to'plamlar soni. Nemitskiy tekisligining oddiy makon emasligi, vaznning hisoblanmasligi va Nemitskiy samolyotining xarakterini hisoblash mumkinligi haqida boshqa dalillar keltirilgan.

**Kalit so'zlar:** topologik fazo; Nemitskiy samolyoti; zichlik, tarmoq og'irligi, psevdokarakter, tarqalish, darajada, mahalliy zich, funktsional va zaif germetiklik, preshansoni, -belgi, Shansoni.

**Аннотация.** В этой статье исследуются кардинальные и топологические свойства плоскости Немыцкого. Известно, что эта плоскость является сепарабельным пространством без счетной базы, поэтому она не является метризуемым пространством. Эта плоскость удовлетворяет первой аксиоме счетности и не удовлетворяет второй аксиоме счетности. Мы представляем некоторые другие интересные свойства плоскости Немыцкого. В этой статье найдены следующие кардинальные инварианты плоскости Немыцкого: теснота, сетевой вес, псевдохарактер, распространение, протяженность, локальная плотная, функциональная и слабая теснота, число Прешанина, -характер, число Шанина, число открытых множеств. Приведены другие доказательства о том, что плоскость Немыцкого не является нормальным пространством, о несчетности веса и о счетности характера плоскости Немыцкого.

**Ключевые слова:** топологическое пространство; плоскость Немыцкого; плотность, вес сети, псевдохарактер, распространение, протяженность, локальная плотная, функциональная и слабая плотность, число Прешанина, -характер, число Шанина.

**MSC:** 22A05, 54H11, 54D30, 54G20.

Let  $L$  be a subset of the plane defined by the condition  $y \geq 0$ , i.e., a closed upper half-plane. Let  $L_1$  denote the line  $y = 0$  and let  $L_2 = L \setminus L_1$ . For each  $x \in L_1$  and  $r > 0$ , let  $U(x, r)$  be the set of all points from  $L$  lying inside the circle of radius  $r$  tangent to  $L_1$  at the point  $x$ . Let further  $U_i(x) = U(x, \frac{1}{i}) \cup \{x\}$ ,  $i = 1, 2, \dots$ . For every  $x \in L_2$  and  $r > 0$ , let  $U(x, r)$  be the set of

all points of  $L$  lying inside a circle of radius  $r$  centre at  $x$ , and let  $U_i(x) = U(x, \frac{1}{i}) \cup \{x\}, i = 1, 2, \dots$  [1].

One can readily check that the collection  $\{B(x)\}_{x \in L}$ , where  $\{B(x)\} = \{U_i(x)\}_{i=1}^\infty$ , has the properties (BP1)-(BP3). The set  $L_1$  is closed with respect to the topology generated by the neighbourhood system of  $\{B(x)\}_{x \in X}$ . The space  $L$  is called the Nemytsky plane.

(BP1). For any  $x \in X$  we have  $B(x) \neq \emptyset$  and for any  $U \in B(x)$  we have  $x \in U$ .

(BP2). If  $x \in U \in B(y)$ , then there exists the set  $V \in B(x)$  such that  $V \subset U$ .

(BP3). For any,  $U_1, U_2 \in B(x)$  there exists  $U \in B(x)$  such that  $U \subset U_1 \cap U_2$  [1].

**Definition 1 [1].** A topological space satisfies the first axiom of separability  $T_1$ , if each of any two distinct points the space has a neighbourhood that does not contain the other of these points, i.e., for any points  $x, y \in X$ ,  $x \neq y$ ,  $U_x, V_x : x \notin U_y, y \notin V_x$ .

**Definition 2 [1].** A topological space satisfies the fourth axiom of separability  $T_4$ , if any two disjoint closed sets topological space have disjoint neighbourhoods.

**Definition 3 [1].** A topological space is called normal if it satisfies the first and fourth axioms of separability, i.e. if the topological space any  $A, B \subset X, A \cap B = \emptyset$  two disjoint nonempty closed sets have disjoint neighbourhoods  $A \subset U, B \subset V$  such that  $U \cap V = \emptyset$ .

The base of topological space is a family of open subsets of a topological space  $X$ , such that any nonempty open set  $G$  is representable as a union of elements of this family.

A family  $B$  of open sets of a  $X$  topological space is a base if and only if, for each point  $x$  of the space  $X$  and its neighbourhood  $U$ , there exists the element  $V$  of  $B$ , such that  $x \in V \subset U$ .

The minimum of the cardinalities of all bases of a space  $X$  is called the weight of the topological space and it is denoted by  $w(X)$

$$w(X) = \min \{|B|, \text{ where } B - \text{base of topological space } X\}.$$

**Definition 4 [1].** The character of a point  $x$  of a topological space  $X$  is the smallest cardinal number of the form  $|B(x)|$ -base at point  $x$  and it is denoted as follows:

$$\chi(x, X) = \min \{|B(x)|, \text{ where } B(x) - \text{base of } x\}.$$

The character of a topological space is defined as follows

$$\chi(X) = \sup \{\chi(x; X) : x \in X\}.$$

**Theorem 1.** Let be  $L$  the Nemytsky plane. Then

- 1.1) the Nemytsky plane is not a normal space;
- 2) the weight of the Nemytsky plane is uncountable;
- 3) the character of the Nemytsky plane is countable.

**Definition 5 [2].** The weak density of topological space  $X$  is the smallest cardinal number  $\tau \geq \aleph_0$ , such that there  $\pi$ -base  $X$  coinciding with  $\tau$  centered systems of open sets, i.e., there is a  $\pi$ -base  $B = \cup \{B_\alpha : \alpha \in A\}$ , where  $B_\alpha$  is a centered system of open sets for each  $|A| = \tau$ .

The weak density of a topological space  $X$  is denoted by  $wd(X)$ . If  $wd(X) = \aleph_0$ , then we say that a topological space  $X$  is weakly separable [2].

A topological space  $X$  is called  $\tau$ -weakly dense, if  $wd(X) = \tau$ .

**Proposition 1.** Weakly density of  $X$  topological space is  $\tau$  if and only if there exists a  $\pi$ -network that coincide with the union of  $\tau$  centered systems of sets.

**Proposition 2.** If  $d(X) = \tau \geq \aleph_0$ , then  $wd(X) \leq \tau$ .

**Proposition 3.** For any topological space  $X$ , we have

$$c(X) \leq wd(X) \leq d(X).$$

**Definition 6 [1].** The smallest cardinal number  $m \geq \aleph_0$  such that every subset of a space  $X$  consisting only of isolated points has cardinality  $\leq m$ , is denoted by  $hc(X)$  or  $s(X)$ , and is called its spread of  $X$ .

$$s(X) = \sup \{|D| : D \subset X \text{ a discrete subset of the space } X\}.$$

**Theorem 2.** Let be the  $L$  Nemytsky plane. Then

- 1) the weak density of  $L$  is countable.
- 2)  $\pi$ -the weight of the Nemytsky space is countable .
- 3) the spread of the Nemytsky plane is uncountable.

**Proof.** 1) The Susnumber and the density of the Nemytsky space are countable, i.e.  $c(L) = \aleph_0$  and  $d(L) = \aleph_0$ , then by proposition 3 we have that the weak density of the space  $L$  is countable  $wd(L) = \aleph_0$ .

2) Now we will prove that  $\pi$ -weight of the Nemytsky space is countable by the two ways. We will use the following theorems.

**Definition 7 [1].** The tightness of a point  $x$  a topological space  $X$  is the smallest cardinal number  $m \geq \aleph_0$  with the following property: if  $x \in [C]$ , then there exists  $C_0 \subset C$  such that  $|C_0| \leq m$  and  $x \in [C_0]$ . This cardinal number is denoted by  $t(x, X)$ . The tightness of the topological space  $X$  is the supremum of all numbers  $t(x, X)$  for  $x \in X$ . This cardinal number is denoted by  $t(X) = \sup \{t(x, X) : x \in X\}$ .

We give other definitions of crowding into points.

**Definition 8 [1].** For any topological space  $X$ , the tightness  $t(X)$  is equal to the smallest cardinal number  $m \geq \aleph_0$  such that for any  $C \subset X$  non-closed set, there exists  $C_0 \subset C$  such that  $[C_0] \leq m$  and  $[C_0] \setminus C \neq \emptyset$ .

**Definition 9 [9].** A family  $N = \{M_i : i \in S\}$  of subsets of a topological space  $X$  is called a network of space  $X$  if, for each neighbourhood  $U$  of a point  $x$ , there is  $i \in S$  such that  $x \in M_i \subset U$ .

The network weight is defined as the smallest cardinal of the form  $|N|$ , where is the  $N$  network  $X$ . Network weight of  $X$  is denoted by  $nw(X)$ :

$$nw(X) = \min \{|N| : N - \text{network of } X\}$$

**Definition 10 [1].** A pseudocharacter  $T_1$  of a space  $X$  at a point  $x$  is defined as the smallest cardinal of the form  $|U|$ , where  $U$  is a family of open  $X$  sets such that  $\bigcap U = \{x\}$ , this cardinal is denoted as the supremum of all cardinals  $\psi(x, X)$ , where is denoted by  $\psi(X) = \sup \{\psi(x, X) : x \in X\}$ .

Note that  $\psi(x, X) \leq \chi(x, X)$  and  $\psi(X) \leq \chi(X)$  for each  $T_1$ -space  $X$  and any  $x \in X$ .

**Definition 11 [9].** The smallest cardinal number  $m \geq \aleph_0$  such that every closed subset of a space  $X$  consisting only of isolated points has cardinality  $\leq m$ . It is called the extent of the  $X$  and it is denoted by  $e(X)$ .

**Definition 12 [4].** We say that the local density of a topological space  $X$  is  $\tau$  at a point  $x$ , if  $\tau$  is the smallest cardinal number such that  $x$  has a neighbourhood of density  $\tau$   $X$ . The local density at a point  $x$  is denoted by  $ld(x)$ . The local density of a topological space  $X$  is defined as the supremum of all numbers  $ld(x)$  for  $x \in X$  :  $ld(X) = \sup \{ld(x) : x \in X\}$

**Theorem 3 [4].** For each topological space  $X$ , we have  $ld(X) \leq d(X)$ .

**Definition 13 [8].** The functional tightness  $t_0(X)$  of a space  $X$  is the smallest infinite cardinal number  $\tau$  such that every  $\tau$ -continuous real-valued function on  $X \setminus A$  continuous.

**Definition 14 [6].** The weak tightness  $t_c(X)$  of a space  $X$  is the smallest infinite cardinal number  $\tau$  such that the following condition is satisfied: if a set  $A \subset X$  is not closed  $X$  then there exist a point  $x \in [A] \setminus A$ , a set  $B \subset A$  and a set  $C \in X$  such that  $x \in [B]$ ,  $[B] \subset [C]$  and  $[C] \leq \tau$ .

**Definition 15 [10].** A family  $\gamma$  of nonempty open sets  $X$  is called a  $\pi$ -base at a point  $x \in X$ , if for each of its neighbourhoods  $Ox$  there exist  $U \in \gamma$  such that  $U \subset Ox$ . At the same time,  $\pi\chi(x, X) = \min \{|\gamma| : \gamma - \pi\text{-base } X\}$  is called  $\pi$ -character of  $X$  at the point  $x$ , and  $\pi\chi(X) = \sup \{\pi\chi(x, X) : x \in X\}$  is called  $\pi$ -character of the space  $X$ .

**Proposition 5 [9].** For any topological space, we have:

$$\pi\chi(x, X) \leq \chi(x, X), \quad \pi\chi(X) \leq \chi(X)$$

**Definition 16 [10].** Number of open sets of the  $X$  topological space:

$$o(X) = |\tau|.$$

If  $\varphi$  is a cardinal invariant, then the hereditary cardinal invariant  $h\varphi$  generated by the cardinal invariant  $\varphi$  is defined as follows:  $h\varphi(X) = \sup\{\varphi(Y) : Y \subset X\}$ .

Hereditary density space of  $X$  defined as follows:

$$hd(X) = \sup\{d(Y) : Y \subset X\}$$

**Theorem 4 [9].** Let  $X$  be a separable space. Then every uncountable cardinal is a caliber of  $X$ .

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## MAXSUSLIKKA EGA BO'LGAN IKKINCHI TARTIBLI DIFFERENTIAL OPERATOR UCHUN SHUTRM-LIUVILL MASALASINI MAXSUS VARIATION AYIRMALI SXEMALAR YORDAMIDA YECHISH

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**Annotatsiya:** Ushbu maqolada maxsuslikka ega bo'lgan Shutrm- Liuvill masalasi uchun aniq variatsion –ayirmali sxema qurib, uning yagonaligi isbotlangan bu ayirmali sxemadan foydalanib, istalgan aniqlikdagi “ $m$ ” – rangli “qirg'ilgan” variatsion ayirmali sxema qurilib, uning xos sonlari va ularga mos keluvchi xos vektorlar bo'yicha yaqinlashish tezligi aniqlangan. Maxsus tengmas qadamli to'rtinchi tanlash natijasida, qurilgan variatsion – ayirmali sxema uchun bo'laklab uzluksiz funktsiyalar sinfidan mumkin bo'lgan eng yuqori aniqlikka erishilgan.